TRANSFORMATION MODELS*

The first two volumes of *Capital* are based on the law of value according to which commodities are exchanged on the basis of the labor-time they embody.[1] Therefore, the value structure of sector-i can be written as

(1) $\Pi_i = C_i+V_i+S_i=C_i+V_i(1+s_i)$, where C_i denotes constant capital, V_i variable capital, S_i surplus value, s_i rate of surplus value and Π_i value of sector-i.

In addition, the rate of profit is defined as

(2) $r_i=S_i/(C_i+V_i)=s_i(1-q_i)$, where q_i denotes the organic composition of capital in sector-i and is defined as

(3) $q_i=C_i/(C_i+V_i)$.

Marx argues that in a capitalist economy there will be

(a) a tendency towards the equalization of the rate of surplus value (due to labor mobility), i.e.,

(4) $s_i=s_j$ for all i's and j's.

(b) a tendency towards the equalization of sectoral rates of profit (due to competition among capitalists and made possible by mobility of capital), i.e.,

(5) $r_i=r_j$ for all i's and j's.

For both tendencies to be effective, a uniform q must prevail all over the economy, i.e.,

(6) $q_i=q_j$ for all i's and j's.

However, one can know from simple observation that in general (6) does not hold in a capitalist economy. But, when

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[1] The term "embodied labor time" means not the "historical cost of a commodity, but rather its current cost of reproduction in labor-time.

(7) $q_i \neq q_j$, for at least some i's and j's,

either (4) or (5) can hold true but not both. Thus, the conclusion that in an economy where a uniform rate of surplus value prevails and where sectoral organic composition of capital is differentiated, rates of profit cannot be equalized if the law of values is in effect. Another alternative is that, if under the postulated conditions there is also a uniform rate of profit, commodities cannot be exchanged at the labor values.

At various places in *Capital*, vol. I (p. 163, 212) Marx points out that the second case is the correct one. However, throughout the first two volumes he ignores this problem by assuming a uniform organic composition of capital, and postpones its solution to the third volume. In other words, according to Marx, in a capitalist economy (4), (5) and (7) will hold and commodities will be exchanged at prices deviating from values.

Marx's solution to the problem was first given in a letter he wrote to Engels in 1862 and later in the third volume of *Capital*. For that purpose, he utilizes the concept of the price of production which is defined as cost price x (1+equalized profit rate), where cost price is composed of constant and variable capital.

For a two-sector economy value and price systems can be written as follows:

(8) <u>Value system</u>

$$C_1+(1+s)V_1=\Pi_1$$
$$C_2+(1+s)V_2=\Pi_2$$

(9) <u>Price of production system</u>

$$(C_1+V_1)(1+r)=P_1$$
$$(C_2+V_2)(1+r)=P_2$$

For the price of production system, the equalized rate of profit is defined as

(10) $R=S/(C+V)$, where $C=C_1+C_2$, $V=V_1+V_2$ and $S=S_1+S_2$.

It is obvious that Marx starts the transformation (of values into prices of production) procedure by the equality

(11) R=S, where R denotes total profits. It follows that

(12) $\Pi_1+\Pi_2=P_1+P_2$.

In other words, in Marx's procedure total profits equal total surplus value (11) and total prices of production equals total values (12)[2]. Equalities (11) and (12) were called "invariance postulates" by Blaug (1968).

On the basis of this procedure Marx reaches two important conclusions:

(a) The direction (sign) of price-value differential in a particular sector will be determined by the relation between the average organic composition of capital and that of the sector in question, and

(b) The transformation of values into prices of production involves a redistribution of surplus value among the sectors. Hence the surplus value appropriated by the capitalists in a particular sector will differ from surplus value created in that sector, as long as the organic composition of that sector differs from the average.

It was argued that Marx's transformation procedure involves certain errors.

(i) Marx computes inputs in terms of values and outputs in terms of prices of production. It was argued that this would result in incorrect prices and rate of profit, and the real wage would be different in the value and price systems. According to Bortkiewicz (1907), this would disturb inter-sectoral flows and the conditions of inter-sectoral equilibrium would be disrupted.

(ii) By using values to compute the rate of profit, Marx defines this concept which belongs to the world of prices (appearances) in

[2] In all of the foregoing Π_i and P_i were treated as totals rather than unit values and prices.

terms of the variables of the value system (substance) and this is not legitimate.

Marx was aware of this situation. However, he deemed it unnecessary to dwell more upon the subject, since he was interested in the sign of the deviation of prices from values while the said imperfections in his procedure were related to the magnitude of deviations.

Furthermore, it may be argued that

(a) Marx's numerical examples upon which this so-called transformation procedure is based do not involve inter-sectoral relations.

(b) As Winternitz (1948) points out, Marx's method is logically consistent if it is interpreted as a dynamic process (but not in accord with capitalist reality). Such an approach to the transformation which views it as a dynamic process was further developed by Shaikh (1973) and Morishima (1974). Both of these authors argue that Marx's solution can be regarded as the first step in an iterative process which transforms values into prices of production.

(c) Again Winternitz (1948) points out that the passage from one valuation scheme to another (from values to prices) would necessarily involve a disruption of inter-sectoral equilibrium. Thus, this is not a shortcoming of Marx's method as was alleged by Bortkiewicz (1907).

All these aside, a static transformation model should obviously evaluate both inputs and outputs in terms of the same valuation scheme. One such model was put forward by Bortkiewicz (1907) in the form of a <u>three-sector simple reproduction model</u> which can be formalized as follows:

(13) <u>Value system</u>

$$C_1+V_1+S_1=\sum C_i$$
$$C_2+V_2+S_2=\sum V_i$$
$$C_3+V_3+S_3=\sum S_i$$

Where C_i, V_i and S_i have the same meanings as on page 1 and where the first sector produces capital (producer) goods, the second wage goods and the third luxury goods. The fact that all surplus value is spent of luxury goods as a result of the assumption of simple reproduction.

Marx's transformation equations for this case can be written as:

(14) <u>Price system</u>

$$C_1+V_1+r(C_1+V_1)=P_1\Sigma C_i$$
$$C_2+V_2+r(C_2+V_2)=P_2\Sigma V_i$$
$$C_3+V_3+r((C_3+V_3)=P_3\Sigma S_i$$

And the so-called invariance postulates as:

(15) $$\sum_{i=1}^{3} r(C_i+V_i)=\sum_{i=1}^{3} S_i$$

(16) $$\Sigma C_i+\Sigma V_i+\Sigma S_i=P_1^*\Sigma C_i+P_2^*\Sigma V_i+P_3^*\Sigma S_i$$

On the other hand, Bortkiewicz's transformation model is as follows:

(17) <u>Price system</u>

$$(1+r)(C_1P_1+V_1P_2)=P_1^*\Sigma C_i$$
$$(1+r)(C_2P_1+V_2P_2)=P_2^*\Sigma V_i$$
$$(1+r)(C_3P_1+V_3P_2)=P_3^*\Sigma S_i$$

where r denotes the equalized rate of profit. Both P_i's in (14) and P_i^*'s in (17) above are coefficients which convert values to prices of production.

Equations in (17) will give us the rate of profit, r, and two relative prices. In order to find absolute prices another equation must be added to (17). Bortkiewicz argues that if both values and prices are to be measured in same units (money commodity), one has to determine in which sector this commodity is produced and has to set its price equal to one. He takes it to be the third one and thus writes the equation

(18) $P_3^*=1$.

Solution of (17) together with (18) will give us r and P_i^*'s. Equation (18) together with the assumption of simple reproduction implies that total profits equal total surplus value. However, total prices will deviate from

total values unless the organic composition of capital in sector-3 happens to be equal to the average. Hence the two invariance postulates will ordinarily not hold together if the economy does not have a particular structure.

Moreover, both the sign and the magnitude of price-value differentials will depend on the technical conditions in the third sector in which the money commodity is produced. Therefore, Marx's conclusion concerning the relation between price-value differentials and the organic composition differences will hold only for relative price-relative value differentials. However, his conclusion concerning the redistribution of surplus value as a result of transformation continues to hold true, and this is the more important of his two conclusions.

Bortkiewicz also reaches the conclusion that the rate of profit is determined by production conditions in the first two sectors whose products enter (directly or indirectly) into the consumption basket of laborers. This he takes to be a proof of that profit is nothing but a deduction from the value created by laborers.

Bortkiewicz's model was criticized on the following grounds:

(a) It complicates the problem by using unnecessarily restrictive assumptions, i.e. simple reproduction, for one.

(b) Samuelson (1970) alleges that by the use of variables P_i^* Bortkiewicz conceals the fact that prices can be derived directly from physical production coefficients without making use of labor values. For instance,

C_i=physical units x value

P_i^*=price/value

$C_iP_i^*$=(price/~~value~~) x physical units x ~~value~~

$C_iP_i^*$=price x physical units

A more general solution to the transformation problem is formulated in Winternitz (1948). His model is more general than Bortkiewicz's in that he

does not assume simple reproduction. Moreover, his solution of the model is mathematically more elegant.

$$(19)\quad C_1P_1^*+V_1P_2^*+R_1=A_1P_1^*=(1+r)(C_1P_1^*+V_1P_2^*)$$

$$C_2P_1^*+V_2P_2^*+R_2=A_2P_2^*=(1+r)(C_2P_1^*+V_2P_2^*)$$

$$C_3P_1^*+V_3P_2^*+R_3=A_3P_3^*=(1+r)(C_3P_1^*+V_3P_2^*)$$

where $\sum C_i \underset{\leq}{\overset{\geq}{=}} A_1$, $\sum V_i \underset{\leq}{\overset{\geq}{=}} A_2$, $\sum S_i \underset{\leq}{\overset{\geq}{=}} A_3$

and $\sum C_i+\sum V_i+S_i=\sum A_i$.

By using the definition of the rate of profit, one can write

$$(20)\quad A_1P_1^*/(C_1P_1^*+V_1P_2^*)=A_2P_2^*/(C_2P_1^*+V_2P_2^*)$$

$$=(1+r)$$

Thus (19) will give us r and two relative prices in terms of, say, P_1^*. In order to calculate absolute prices Winternitz adds the following equation:

$$(21)\quad \sum A_1=\sum A_1P_1^*,$$

which is nothing but the postulate that total values equal total prices.

While Winternitz's model is simpler, it is still very restrictive since it assumes that the use of a product is determined by its origin. In other words, every product can only be utilized in a specific way. However, May (1948) argued that Winternitz's model can very easily be extended into an n-sector model. Such an n-sector transformation model was first put forward by Seton (1957).

First a few definitions:

k_{ij} — the amount of the j-th sector's product which is used in the i-th sector both as input ("machine feeding") and as consumption good by the workers ("labor feeding") in that sector; k_{ij} is measured in labor values.

The sectoral distribution of the j-th sector's product can be shown as

$$(22) \quad \sum_{i=1}^{n} k_{ij}+e_{ij}=a_j,$$

where e_j shows the capitalists' consumption of the j-th product and a_j the total output of the j-th sector. e_j denotes not only consumption but also accumulation (amount of j-th product used as investment good) if there is extended reproduction.

The cost structure of the i-th sector is written as

$$(23) \quad \sum_{i=1}^{n} k_{ij}+S_i=a_i$$

where Si denotes the surplus value created in sector-i.

The price system is

$$(24) \quad Kp=gA^{*}p$$

where $K=[k_{ij}]$; A^{*} is a diagonal matrix whose elements are a_i's; p is a column vector of unit prices, p_i. Seton defines $g=(1+r)$ as cost ratio, and r is defined as $r=R_i/a_ip_i$.

Obviously, any p which will equalize r will also equalize g and *vice versa*.

If one makes the following definitions

$a_{ij}=k_{ij}/a_i$ and $A=[a_{ij}]$,

one can write

$$(25) \quad Ap=gIp$$

$$(26) \quad (A-gI)p=0.$$

For (26) to have a non-trivial solution so that $p\neq0$, the following equality must hold:

$$(27) \quad |A-gI|=0.$$ [3]

[3] The equalized cost ratio of g turns out to be the Perron-Frobenius root of the augmented technology matrix A.

(27) implies that the vectors in (26) are linearly dependent. Thus when 25 [or (26)] is solved together with (27), we will get n-1 relative prices and the g (also r). Seton argues that the condition of equal profitability will get us this far. In order to find absolute prices one can adopt any of the invariance postulates.

Samuelson (1970) introduced a new transformation model. Here we have n sectors and one type of homogenous labor-power where

a_{0j} the amount of direct labor input required for the production of one unit of good-j;

a_0 row vector of direct labor requirements (1xn);

a_{ij} amount of the i-th good required for the production of one unit of good-j;

A square Leontief (input-output) matrix showing the intermediate goods requirements (nxn);

m_i the amount of good-i which is needed for production and reproduction of one unit of labor;

m the column vector of minimum subsistence goods needed as real wage;

w wage rate (a scalar);

s rate of surplus value;

Π the row vector of unit labor values (1xn).

The value system can be written as

(28) $\Pi = wa_0 + \Pi A + swa_0 = wa_0[I-A]^{-1}(1+s)$

(29) $\Pi m = w$

(28) and (29) will give us n-1 relative values together with s and w.

If p is the row vector of prices (1xn), the price system can be written as

(30) $p = [wa_0 + pA](1+r) = wa_0(1+r)[I-A(1+r)]-1$

(31) $pm = w$.

Here (30) and (31) will give us n-1 relative prices together with w and r. However, if wage rate in the price system is taken to be equal to that of the value system, in other words if it remains invariant, and is given, then (30) and (31) will give us absolute prices.

According to Samuelson, the transformation algorithm is nothing but a process of "erase and replace," since when physical input coefficients are given, it is possible to find prices independently of values. Thus, he argues that the labor theory of value is nothing but an "unnecessary detour" since prices are in no sense determined by labor values (Samuelson, 1971, 1974).

Moreover, he rightly points out that the solution of the price system "involves an n-th degree polynomial for the appropriate positive root r*, whereas" the solution of the value system "involves solving only a linear equation for s* (Samuelson, 1970, p. 423). Later he advanced the argument that Marx started with the value system since its solution is easier (Samuelson, 1971, p. 418).

Morishima (1974) interprets the transformation as a dynamic process which transforms the values of the simple commodity production into the prices of capitalist mode of production. He argues that this is an iterative process which can be started with values as Marx did. If y_e is the long-run balanced growth output vector and if Π denotes the vector of values, and p_e the equilibrium price vector, then when the production realized is y_e,

(32) $\Pi y_e = p_e y_e$ (total value=total price)

(33) $R y_e = S_e y_e$ (total profit=total surplus value)

Where R is the vector of profits per unit of output and S_e the vector of surplus value per unit of output.

According to Morishima this iterative process is convergent, i.e., starting with values one will end up with the "true" and "correct" prices of production. Therefore, Marx's transformation procedure constitutes the first step in the right direction. Morishima reaches the conclusion that

Marx did not have the proper mathematical tools to deal with such a complicated problem. As a matter of fact, he points out that some of these tools were yet to be discovered and that it is surprising that Marx came so near to solving it.

In Akyüz (1976, 1977), there is a two-sector economy where the first one produces the means of production and the second the consumption goods, unit production functions can be written as follows:

$$(34)\quad \begin{aligned} a_1\lambda_1+\alpha_1&=\lambda_1 \\ a_2\lambda_1+\alpha_2&=\lambda_2 \end{aligned}$$

where a_i shows the amount of the means of production used up in the production of one unit of good-i; α_i the amount of labor-time required for the production of one unit of good-i; and λ_i the labor value of the i-th good. If X denotes the physical output of the first sector and Y that of the second sector, total production equations can be written as follows:

$$(35)\quad \begin{aligned} Xa_1\lambda_1+X\alpha_1&=X\lambda_1=Q_1 \\ Ya_2\lambda_1+Y\alpha_2&=Y\lambda_2=Q_2 \end{aligned}$$

where Q_i is the total value produced in sector-i.

Then constant and variable capital in sector-i (C_i and V_i respectively) are given by the following equations:

$$(36)\quad C_1=Xa_1\lambda_1 \qquad C_2=Ya_2\lambda_1$$

$$(37)\quad V_1=X\alpha_1 w_s\lambda_2 \qquad V_2=Y\alpha_2 w_s\lambda_2$$

Where w_s shows the amount of consumption good which is required for reproduction of one unit labor-time (value of labor-power, i.e., real wage).

The surplus value is defined as

$$(38)\quad \begin{aligned} S_1&=X\alpha_1-V_1=X\alpha_1(1-w_s\lambda_2) \\ S_2&=Y\alpha_2-V_2=X\alpha_2(1-w_s\lambda_2) \end{aligned}$$

and the rate of surplus value, s, as

$$(39)\quad s=S_i/V_i=(1-w_s\lambda_2)/(w_s\lambda_2).$$

Then we can rewrite (35) in the form

(40) $C_1+V_1(1+s)=Q_1$

$C_2+V_2(1+s)=Q_2$.

Akyüz argues that in order to be comparable, both the values and the prices must be measured in the same units: the value of a commodity is measured by the amount of labor-time embodied in it and its price by the amount of labor-time it can purchase. Let us assume that p_i^m shows the price of commodity-i in money terms; w^m the wage rate in money terms. Then,

(41) $p_i=p_i^m/w^m$,

where Pi denotes the price of commodity-i in terms of labor-time which one unit of that commodity can purchase.

Thus, the price equivalent of (34) can be written as follows:

(42) In money terms

$(a_1p_1^m+\alpha_1w^m)(1+r)=p_1^m$

$(a_2p_1^m+\alpha_2w^m)(1+r)=p_2^m$

and after dividing through by w^m,

(43) In terms of labor-time

$(a_1p_1+\alpha_1)(1+r)=p_1$

$(a_2p_1+\alpha_2)(1+r)=p_2$.

Since the real wage, $w_s=w^m/p_2^m$, it follows that

(44) $p_2=1/w_s$.

This simply means that the wage in (42) is equal to one. Obviously (44) fixes the unit of measurement for (43).

Given the physical input coefficients and the real wage, (34) and (39) will give us λ_i's and the rate of surplus value, s; and (43) together with (44) will give us p_i's and the rate of profit, r. A comparison of the value and the

price systems shows that while the values are determined independently of income distribution, the prices are not.

Moreover, Akyüz points out that the solution of the price system is independent of the values. However, when the rate of surplus value is given rather than the real wage, the solution of the price system will require prior solution of the value system. Also, by definition $p_i>\lambda_i$ which implies that Marx's conclusion concerning price-value differentials does not hold. However, this is not important since the more significant and important of the conclusions summarized on page 3 is the second one and it remains valid whichever transformation algorithm we happen to use.[4]

Tüzün (1971) argues that in general the transformation models involve the production of all commodities, except one, namely the labor-power. Where this is not so, it is generally assumed that there is only one kind of labor (as Samuelson and Akyüz do). However, it is possible to formulate a general model in which there are n goods and m kinds of labor-power. Before getting into the details of such a model, let us first make some definitions:

a_{ij}	the amount of the $_i$'th good required for the production of one unit of good-j
$A=[a_{ij}]$	the square Leontief matrix showing the intermediate goods requirements (nxn)
b_{ij}	the amount of the i-th kind of labor-power required for the production of good-j
$B=[b_{ij}]$	mxn matrix showing the labor-input requirements
m_{ij}	the amount of the i'th good which is required for the reproduction of one unit of labor-power type-j

[4] Akyüz's equations (43) and (44) can be seen as a special case of Samuelson's equations (30) and (31), since the former assumes only two sectors and one wage good. However, it must be remembered that they measure prices in different units.

$M=[m_{ij}]$ nxm matrix showing the commodity input requirements for the reproduction of different kinds of labor-power

$w=[w_j]$ the row vector of wages (values) of m kinds of labor-power (1xm)

$\Pi=[\Pi_j]$ the row vector of unit labor values (1xn)

s the rate of surplus value

The value system can be written as

(45) $\Pi=wB+\Pi A+swB=(1+s)wB[I-A]^{-1}$

(46) $\Pi M=w$

(45) and (46) constitute the generalized versions of (28) and (29) of Samuelson. The value system of (45) and (46) will give us n+m-1 values and wages (relative) and the rate of surplus value. In order to find absolute values and wages (in terms of labor-time), let us assume that the first kind of labor-power is simple labor-power and measure values and wages in units of this type of labor. Then we can write

(47) $w_1(1+s)=1$

which can be rewritten as

(48) $s=(1-w_1)/w_1=(1-\Pi m_{i1})/\Pi_{mi1}$.

Obviously (48) does also gives us the definition of the rate of surplus value.

The system of equations (45), (46) and (48), when solved, will give us Π_j's, w_j's and the rate of surplus value, s.

Let us substitute (45) into (46)

(49) $\Pi M=(1+s)wB[I-A]^{-1}M$.

If we first multiply and then divide both sides by w_1, and use (48),

(50) $\Pi Mw_1=w_1(1+s)wB[I-A]^{-1}M$

(51) $\Pi M=(w/w_1)B[I-A]^{-1}M$

(52) $\{\Pi-(w/w_1)B[I-A]^{-1}\}M=0$, which implies

(53) $\Pi=(w/w_1)B[I-A]^{-1}$.

The equation (53) means that values measured in terms of simple labor-time are equal to embodied labor-time, since $(w/w_1)B[I-A]^{-1}$ shows total labor requirements, both direct and indirect. Here w/w_1 is a row vector whose elements are w_j/w_1's which are nothing but the coefficients used to convert the j-th kind of labor-power to simple labor-power. Thus, the standard of value is the simple labor-time.

If p is the row vector of prices (1xn), the price system can be written as

(54) $p=(\omega B+PA)+(1+r)=\omega B(1+r)[I-(1+r)A]^{-1}$

(55) $pM=\omega$,

ω being the row vector of money wages.

The solution of (54) and (55) will give us n+m-1 p_j's and ω_j's and the rate of profit, r. In order to find absolute prices and wages, we can, following Akyüz, write

(56) $\omega_1=1=pM_{i1}$,

which is equivalent to (44).

Or we can, following Samuelson (1970), write

(57) $w_1=\omega$, which implies

(58) $\Pi M_{i1}=pM_{i1}$.

The relation between Π and p will depend on which of the equations (56) and (57) we adopt to find absolute prices. If (56) is adopted, then $p_j>\Pi_j$, for all j's, given s>0. If s=0, then $w1=\omega_1=1$; thus $p=\Pi$. If, on the other hand, (57) is adopted, $p\neq\Pi$ as long as sectors have different capital structures. As a result, $p_j>\Pi_j$ for some j's and $p_j<\Pi_j$ for some j's.

This brings us to the matter of invariance postulates. The equality of total values to total prices of production can be written as

(59) $\Pi a=pa$,

where a is a column vector (nx1) whose elements, a_i, are the output level of each sector.

The equality of total surplus value to total profits, on the other hand, can be written as

(60) $\Pi(a-Aa-Mb)=p(a-Aa-Mb)$

where b is a column vector (mx1) whose elements, b_i, are the amounts of different kinds of labor-power used for the production of a. Then, in Seton's terminology, Aa shows "machine feeding" and Mb "labor feeding". Thus (a-Aa-Mb) is the surplus product vector. If we set

(61) $c=Aa-Mb$, c being a nx1 column vector,

(60) can be rewritten as

(62) $\Pi(a-c)=p(a-c)$.

(60) and (62) imply, respectively,

(63) $[\Pi-p]a=0$
(64) $[\Pi-p][a-c]=0$,

which are nothing but invariance postulates. When the condition in (63) is fulfilled, for (64) also to hold true, we may have

(65) $[\Pi-p]=0$, that is $\Pi=p$.

If (63) is satisfied while $p\neq\Pi$, we have from (63) and (64),

(66) $[\Pi-p]c=0$.

In this case, as a possibility, let us assume that

(67) $c=\chi a$,

where the scalar $\chi\leq1$. If $\chi=1$, then c=a, which means that there is no surplus product and hence no surplus value. We know from what we said on page 13 that in this case $p=\Pi$ also. Under these conditions, then, both 63 and 64 will be satisfied.

Another possibility arises is $\chi<1$. If the consumption of capitalists is nil, (67) will imply that a fixed proportion of each sector's output will be used for capital accumulation in physical terms. In other words, the sectoral makeup of a-c is the same as that of a. Thus, while there is capital accumulation, the sectoral structure does not change. The economy is on the balanced growth path. This conclusion is similar to that of Morishima (1974) summarized in equations (32) and (33).

BIBLIOGRAPHY

Akyüz, Y., "A Note on the Marxian Transformation Problem and Income Distribution," *Australian Economic Papers*, June 1976, pp. 96-108

------,*Sermaye, Bölüşüm, Büyüme*, AÜ SBF Yayını, Ankara 1977

Blaug, M., Economic Theory in Retrospect, rev.ed., Richard D. Irwin. Inc, Homewood 1968.

Bortkiewicz, L. von, "On the Correction of Marx's Fundamental Theoretical Construction in the Third Volume of Capital," in Eugen von Böhm-Bawerk, *Karl Marx and the Close of His System*, P.M. Sweezy (ed.), Augustus M. Kelley, New York 1949, pp. 197-221

Marx, K., *Capital*, vols. I & III, Lawrence and Wishart, 1974

------- and F. Engels, *Selected Correspondence*, Progress Publishers, Moscow 1975, pp. 120-123

May, K., "Value and Price of Production: A Note on Winternitz's Solution," *The Economic Journal*, vol. 58, December 1948, pp.596-599

Morishima, M., "Marx in the Light of Modern Economic Theory," *Econometrica*, v. 42, n. 4, July 1974, pp. 611-632

Samuelson, P.A., "The 'Transformation' from Marxian 'Values' to Competitive 'Prices': A Process of Rejection and Replacement," *Proceedings of the National Academy of Sciences*, Sept. 1970; reprinted in *The Collected Scientific Papers of Paul A. Samuelson*, R.C. Merton (ed.), v. 3, The MIT Press, Cambridge 1972, pp. 309-311

------,"Understanding the Marxian Notion of Exploitation: A Summary of the So-Called Transformation Problem Between Marxian Values and Competitive Prices," *The Journal of Economic Literature*, v. 9, n. 2, June 1971, pp. 399-431; reprinted in Merton, *op.cit.*, pp.276-308

------,"Insight and Detour in the Theory of Exploitation: A Reply to Baumol," The Journal of Economic Literature, v. 12, n. 1, March 1974, pp. 62-70

Seton, F., "The 'Transformation Problem'," *Review of Economic Studies*, v. 25, June 1957, pp. 149-160

Shaikh, A., Theories of Value and Theories of Distribution, Unpublished Ph.D. Dissertation, Columbia University, 1973

-----, "Marx's Theory of Value and the 'Transformation Problem'," in *The Subtle Anatomy of Capitalism*, Schwartz, J. (ed.), Goodyear Publishing Co., Santa Monica, 1977; http://minibiblionet.free.fr/textes.page6S/1977.shaikh.pdf

Tüzün, G., *The Transformation Problem*, Unpublished Ph.D. Dissertation, New York University, 1971

Winternitz, J., "Values and Prices: A Solution of the So-Called Transformation Problem," *The Economic Journal*, v. 58, June 1948, pp. 276-280